Variable Structure Control Design for SISO Process: Sliding Mode Approach

A. Saikeerthana*, V. Alamelumangai

Department of Electronics and Instrumentation, FEAT, Annamalai University.

*Corres.author: keerthuashok@gmail.com

Abstract: The industrial processes are truly non-linear by nature. Here a control system design should not limit the operating region of the controller. In this paper an attempt is made to design a sliding mode controller for a non-linear process and analyze the effect of linearization and modeling error present in the model of the process. The performance of the process with sliding mode controller is evaluated using the performance indices settling time, maximum overshoot and steady state error. The MATLAB SIMULINK software is used to simulate the control strategy.

Keywords: FOPTD model, Non-linear control design, sliding mode control, Variable structure control.

1. Introduction

A control system is an interconnection of components forming a system configuration that will provide a desired system response. Most control systems in use today are linear control systems. The most common type is the PID controller, which is adequate in many applications. But non –linear system analysis is more difficult. Mathematically, this is reflected in two aspects. First, non-linear equations cannot in general be solved analytically and therefore a complete understanding of the behavior of a non-linear system is very difficult. Second, powerful mathematical tools like Laplace and Fourier transforms do not apply to non-linear systems. As a result, there are no systematic tools for predicting the behavior of non-linear systems, nor are there systematic procedures for designing non-linear control systems. Instead, there is a rich inventory of powerful analysis and design tools, each best applicable to particular classes of non-linear control problems. Two major and complementary approaches dealing with model uncertainty are Robust Control and Adaptive Control. The goal of robust system design is to retain assurance of system performance in spite of model inaccuracies and changes. A simple approach to Robust Control is the so-called Sliding Control methodology. Sliding mode control plays an important role because it not only can stabilize the system but also provide the capability of disturbance rejection and insensitivity to parameter variations[1]. To control nonlinear processes, the combination of use of differential geometric approach and sliding mode strategy has been proven to be a promising way to the robust control and many advanced SMC schemes have been developed in[2].

In this work an attempt is made to design a continuous-time sliding mode controller for a non-linear chemical process and the performance is evaluated.

2. Process Description

An isothermal chemical reactor shown below is taken to study the performance of sliding mode controller. In this process, the product concentration is controlled by manipulating the feed flow rate, which changes the residence time for constant volume reactor[3].
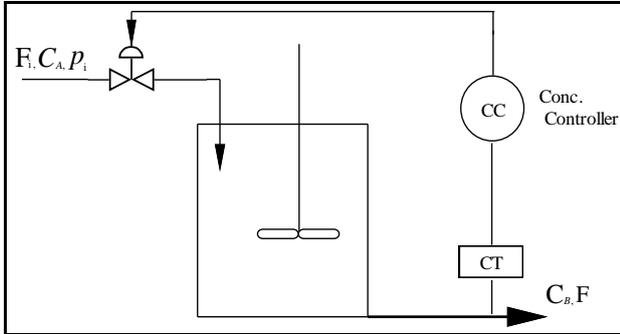

Fig. 1 Schematic of Isothermal chemical Reactor

The Van de-vusse reaction given in the following Equation is under consideration and the desired product is the component B

$$A \rightarrow B \rightarrow C; 2A \rightarrow D \tag{1}$$

For reactor model overall mass balance Equation is given by

$$\frac{d(\rho V)}{dt} = \rho_i F_i - \rho F \tag{2}$$

Where V is the volume in liter, Fi is feed flow rate and F is output flow rate in *liter/min*, and ρi and ρ are the feed flow density and output flow density respectively.

The component material balance of A is given

$$\frac{d(C_A V)}{dt} = C_{Ai} F_i - C_A F + r_A V \tag{3}$$

where C_A is concentration of component A in g mol/liter, C_{Ai} is the concentration of component A in feed flow in g mol/liter and r_A represents generation of species of A per unit volume. It is given by the Equation

$$r_A = -K_1 C_A - K_3 C_A^2 \tag{4}$$

Where K_i and K_j are the reaction rate constant of Equation (1), from the Equation (3),

$$\frac{d(C_A V)}{dt} = V \frac{dC_A}{dt} + C_A \frac{dV}{dt} \tag{5}$$

Hence Equation (3) written as

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - K_1 C_A - K_3 C_A^2 \tag{6}$$

Then, component material balance for **B** is given by

$$\frac{d(C_B V)}{dt} = -C_B F + r_B V \tag{7}$$

Where C_B is the concentration of component B in g mol/liter and r_B is generation of species of B per unit volume, which is given by

$$r_B = K_1 C_A - K_2 C_B \tag{8}$$

Where K_2 is the reaction rate constant for the equations (1), (2) and (7) can be written as

$$\frac{dC_B}{dt} = \frac{F_i}{V} C_B + K_1 C_A - K_2 C_B \tag{9}$$

Thus model consists of three differential equations therefore three state variables.

Often other simplifying techniques are made to reduce the number of differential Equations to make them easier to analyze and faster to solve. Assuming constant volume, resulting differential Equations governing the isothermal chemical reactor are given by following Equations

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - K_1 C_A - K_3 C_A^2 \tag{10}$$

$$\frac{dC_B}{dt} = \frac{F_i}{V}C_B + K_1 C_A - K_2 C_B \tag{11}$$

Here we consider *F/V=D* as the manipulated variable/input, C_A and C_B as state variables, C_{Ai} as disturbance input and C_B as output variable[4].

3. Controller Design

    The SMC design is composed of two stages[5,6,7]. A sliding surface on which the process dynamics is restricted. Subsequently, a feedback control law such that any system trajectory outside the sliding surface is driven to reach the surface in a finite time and keep on it. This therefore makes the closed-loop SMC system robust to matched uncertainties and external disturbances.

    Designing the sliding mode controller, is to choose the sliding surface, *s(t)* to represent a desired global behavior for tracking performance. The *s(t)* selected in this work is an integro-differential equation acting on tracking error expression[2]. The schematic diagram of sliding mode controller is shown in Fig. 2.
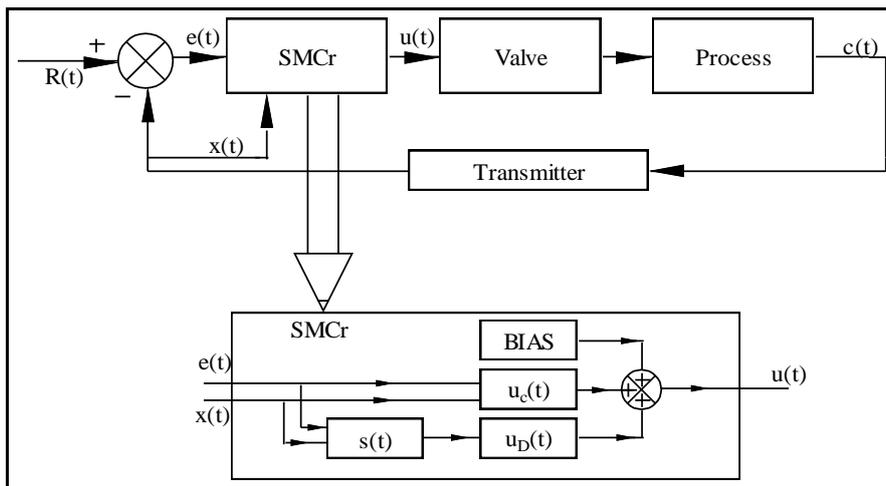

Fig.2 schematic diagram of sliding mode controller

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t)dt \tag{12}$$

Where e(t) is tacking error, λ is tuning parameter and n is the order of the system. The objective is to force the state (error) to move on switching surface s(t) =0.
The second step is to design the control law which drives the controlled variable to its reference value.
The complete SMC control law, u(t) is given by Equation (13)

$$u(t) = u_c(t) + u_D(t) \tag{13}$$

Where $u_D(t)$ incorporate a non-linear element that includes the switching element of control law given by Equation (14). Hence from Equation (13) complete control law will be

$$u(t) = \frac{t_0 \tau}{K}\left\{\lambda^2 e(t) + \frac{x(t)}{t_0 \tau}\right\} + K_D \frac{s(t)}{|s(t)| + \delta} \tag{14}$$

And the sliding function is

$$s(t) = sign(K)\left[\frac{-dx(t)}{dt} + 2\lambda e(t) + \lambda^2 \int_0^t e(t)dt\right] \tag{15}$$

To overcome the drawback (offset error) present in sliding mode controller the sliding surface is replaced by the PID algorithm and the controller parameters can be converted into the tuning parameters of the sliding surface s(t), as below:

$$K_C = \lambda_1$$

$$\tau_I = \lambda_1 / \lambda_0$$

$$\tau_D = (\lambda_1)^{-1}$$

4. Results and Discussion

Isothermal process is modeled in three different operating regions and the corresponding open loop responses are presented in Fig. 3, 4 and 5 respectively. The models obtained are given below:
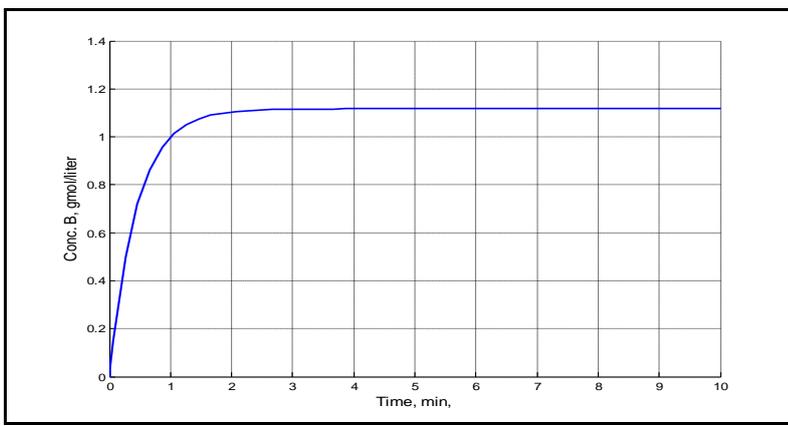
Fig. 3 Open loop response of the Isothermal process for the change in input (*F/V*) from 0 to 0.5714 min^{-1}
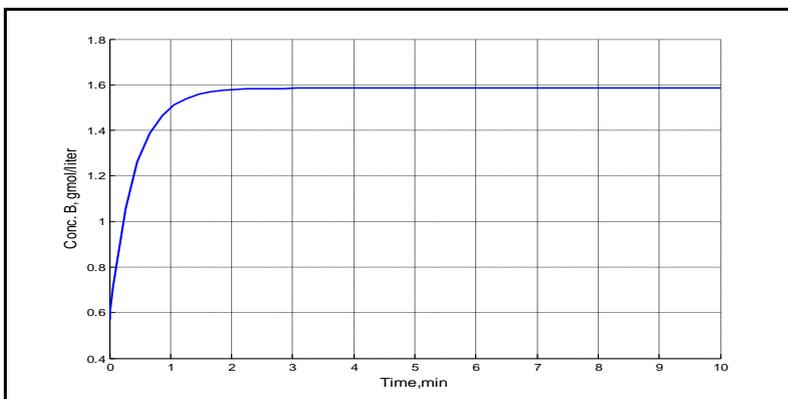

Fig. 4 Open loop response of the Isothermal process for the change in input (*F/V*) from 0.5714 to .8min^{-1}
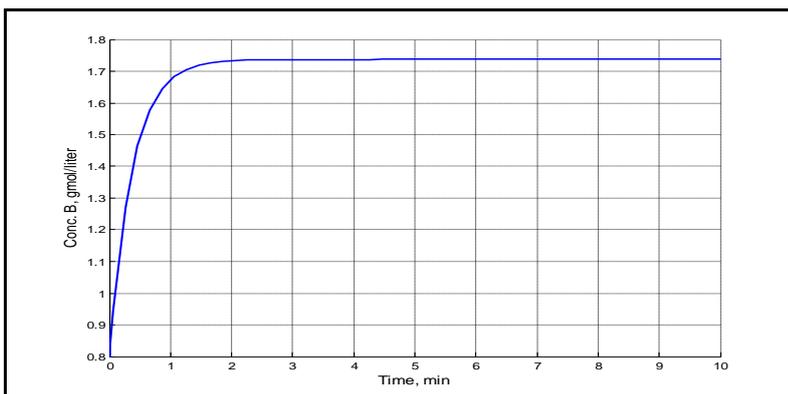

Fig. 5 Open loop response of the Isothermal process for the change in input (*F/V*) from 0.8 to 1.0 min^{-1}

$$G_1(S) = \frac{1.95e^{-0.01s}}{0.46s + 1}$$

$$G_2(S) = \frac{3.4e^{-0.02s}}{0.24s + 1}$$

$$G_3(S) = \frac{4.68e^{-0.003s}}{0.38s + 1}$$

The sliding mode controller and PID based sliding mode controller are designed for three models. Z-N tuned PI controller is also designed for the models for comparison.

The closed loop responses for set point tracking as well as load rejection with PI controller, Sliding mode controller and PID based sliding mode controller for $G_1(S), G_2(S)$ and $G_3(S)$ are presented from Fig. 6 to Fig. 11 respectively. The performance of the process with all the three controllers are evaluated using peak overshoot, settling time and ISE, are presented in the Tables 1, 2 and 3.

Model 1

Operating point:

Input - 0 to 0.5714 min^{-1}

Output – 0 to 1.117 g mol/liter

Model 2

Operating point:

Input - 0 .5714 to 0.8 min^{-1}

Output – 0.5714 to 1.585 g mol/liter

Model 3

Operating point:

Input - 0 .8 to 1 min^{-1}

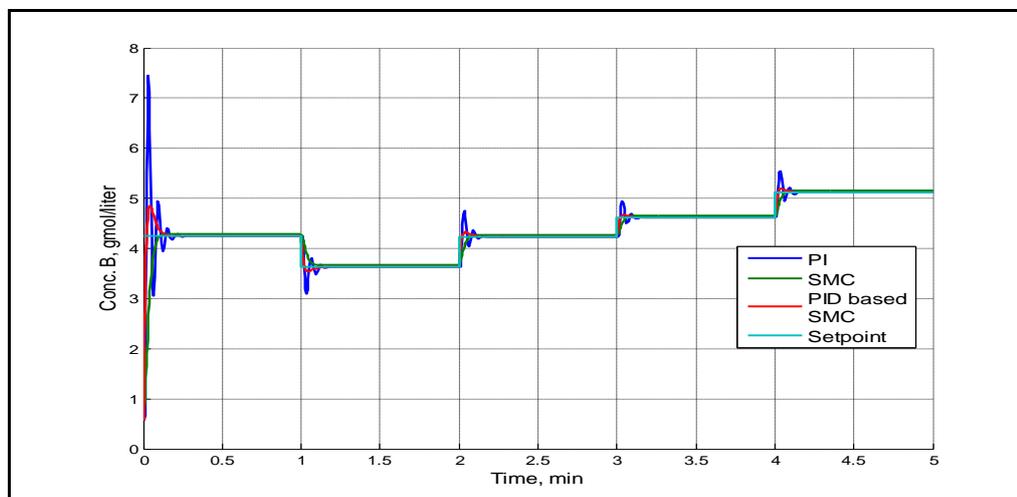Output – 0.8 to 1.737 g mol/liter

Fig. 6 Servo response of the process with PI, SMC and PID based SMC for the model 1
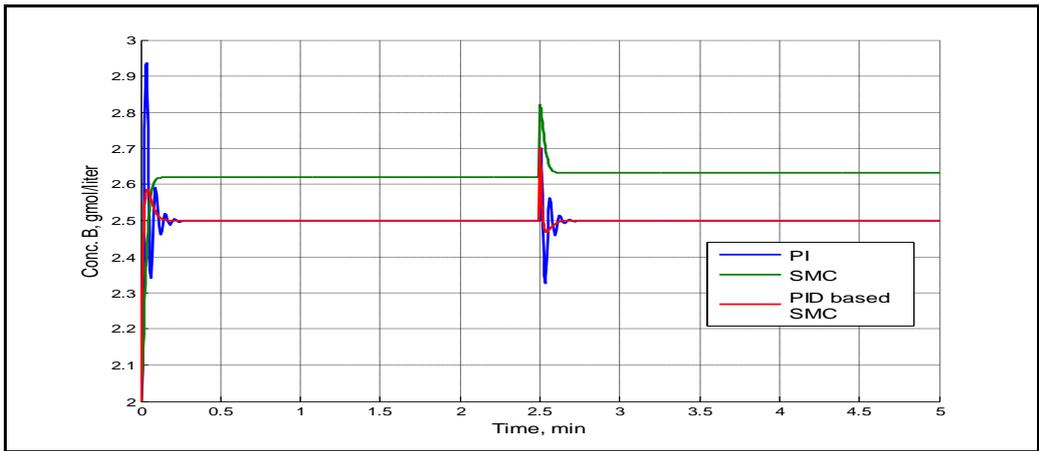
Fig. 7 Regulatory response of the process with PI, SMC and PID based SMC for model 1
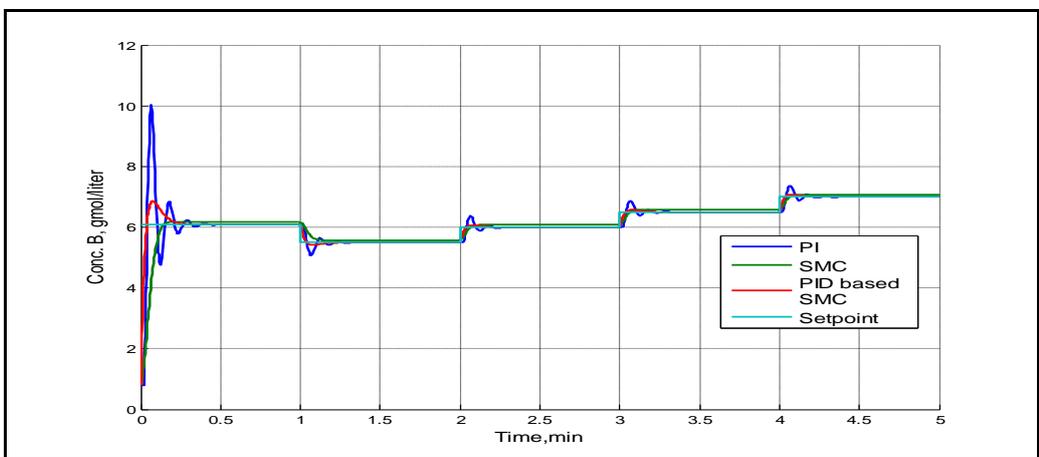

Fig. 8 Servo response of the process with PI, SMC and PID based SMC for the model 2
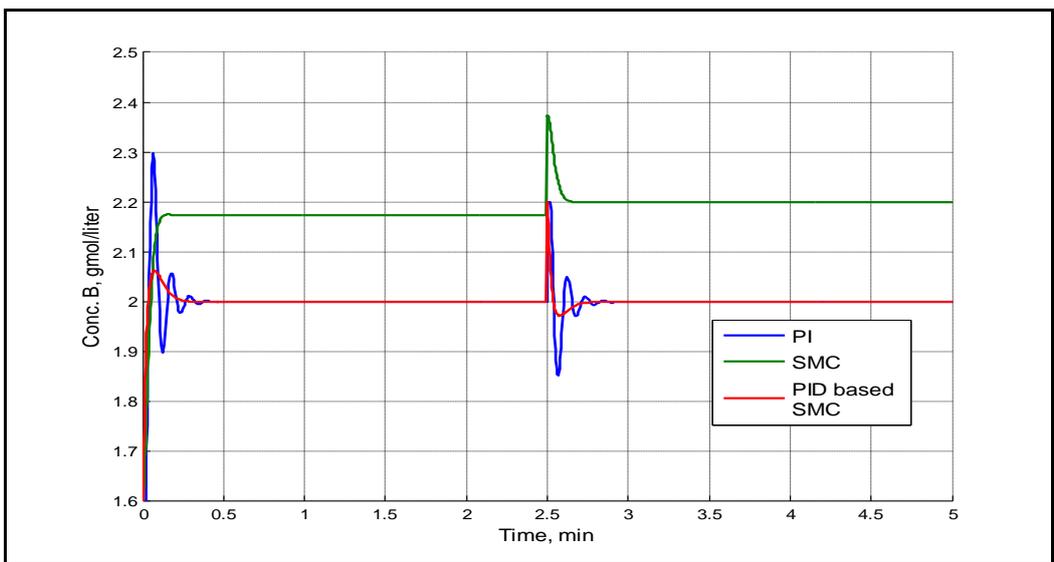

Fig. 9 Regulatory response of the process with PI, SMC and PID based SMC for model
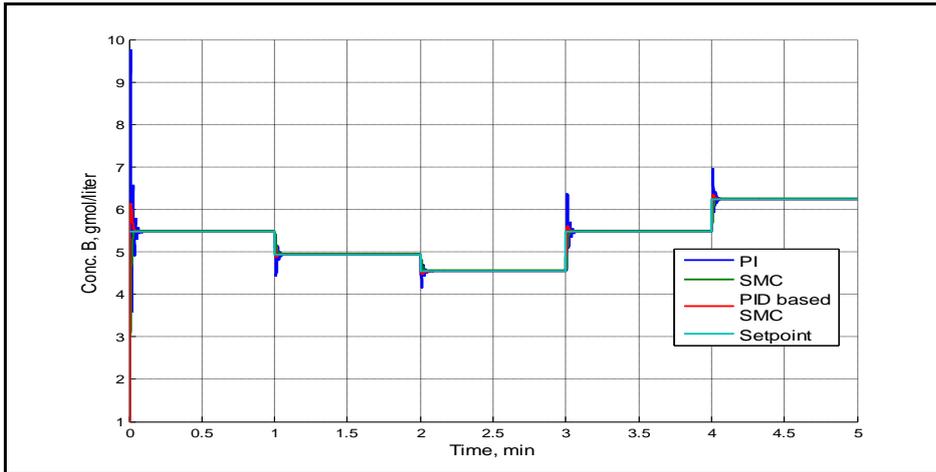
Fig. 10 Servo response of the process with PI, SMC and PID based SMC for the model 3
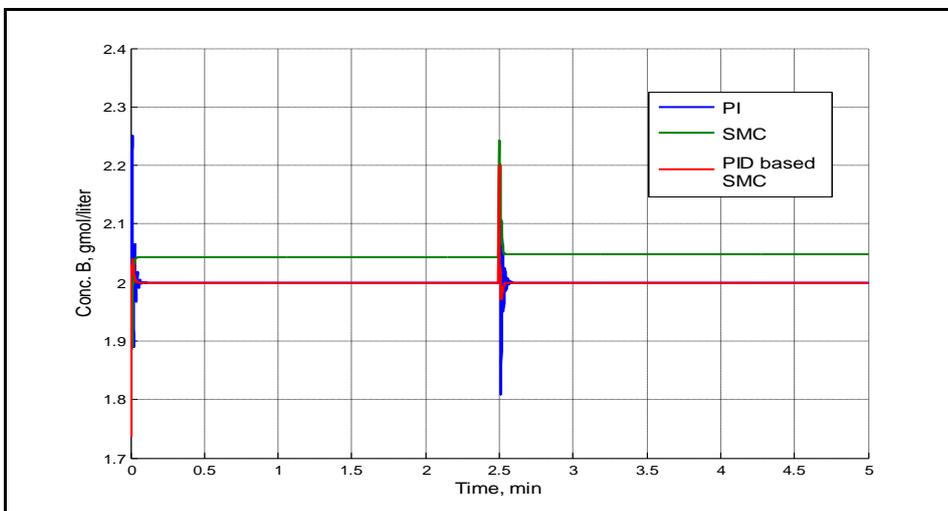

Fig. 11 Regulatory response of the process with PI, SMC and PID based SMC for model 3

The closed – loop response of the process using all the models with the PID based sliding mode controller designed using model 1 alone for multiple change in set point is presented in Fig. 12. The same response is given for PI controller also in Fig. 13.
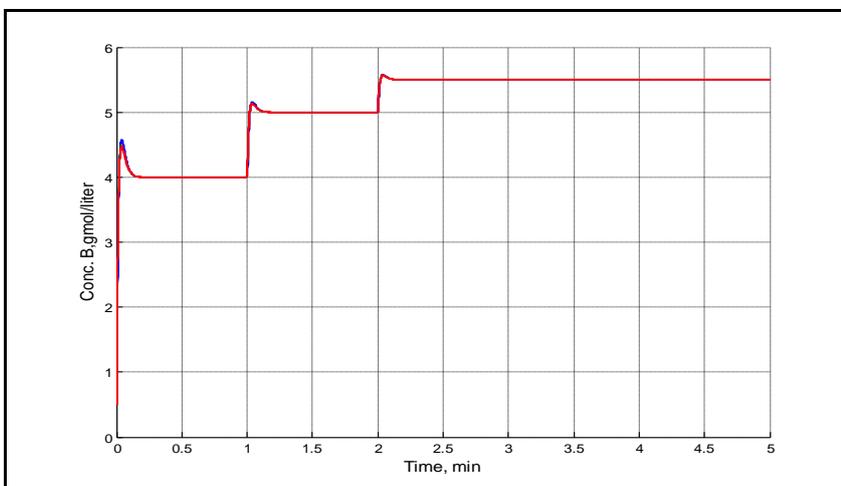

Fig. 12 Servo response of the three models of the process with PID based sliding mode controller designed for model 1
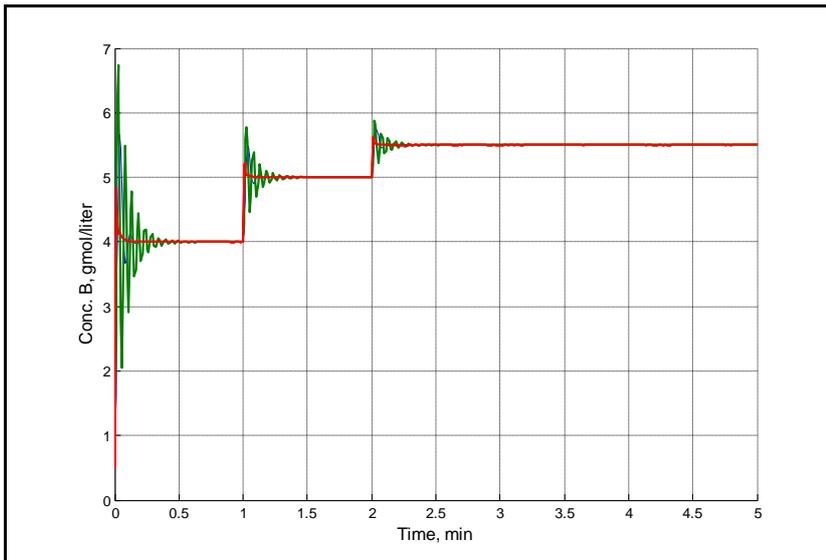
Fig. 13 Servo response of the three models of the process with PI controller designed for model 1

Table 1 Performance indices of the process with PI and PID based sliding mode controller for model 1

Controllers	Settling Time	Max Overshoot	ISE
PI	0.32	1.373	0.0050
PID based SMC	0.14	1.07	0.0012

Table 2 Performance indices of the process with PI and PID based sliding mode controller for model 2

Controllers	Settling Time	Max Overshoot	ISE
PI	0.5	1.15	0.0018
PID based SMC	0.31	1.031	0.00042

Table 3 Performance indices of the process with PI and PID based sliding mode controller for model 3

Controllers	Settling Time	Max Overshoot	ISE
PI	0.14	1.96	0.0094
PID based SMC	0.05	1.14	0.0015

Conclusion

The non-linear chemical process is modelled in three different operating region using process reaction curve method. Sliding mode controller, PID based sliding mode controller and Z-N tuned PI controller are designed for three different models. The performance of the process with PI controller and PID based sliding mode controller are evaluated and presented. The servo response of the three models using PID based sliding mode controller and Z-N tuned PI controller designed for model 1 are also presented. It is observed from the responses presented that the sliding mode controller introduces offset. The performance of the PID based sliding mode controller is better than the Z-N tuned PI controller for all the three models. PID based sliding mode controller designed for model 1 produces good performance for all the models compared with Z-N tuned PI controller. Hence the robustness of the controller is validated.

References

1.   J. Y. Hung, W. Gao and J. C. Hung. "Variable Structure Control: A Survey", IEEE Trans. On Industrial Electronics, 1993, 40, 2-22.

2.   J.J Slontine and W.Li, Applied nonlinear control, Prentice Hall, New Jersy, 1991.
3.   B.B.Musmade, R.K.Munje, B.M.Patre, Design of Sliding mode controller chemical processes for improved performance, International Journal of Control and Automation, 2011, 4.
4.   B. W. Bequette, "*Process Control: Modeling, Design and Simulation*", Prentice Hall of India, 2003.
5.   Camacho and C. A. Smith, Application of sliding mode control to nonlinear chemical processes with variable deadtime, Proceedings of 2nd Congress of Colombian Association of Automatics, Colombia, 1997, 122-128.
6.   V. I. UTKIN, Sliding Mode Control Design Principles and Applications to Electric Drives, IEEE *Tran*s. *Ind. Electr*, 1993, 40, 23– 36.
7.   Y. B. Shtessel, Non-linear Output Tracking in Conventional and Dynamic Sliding Manifolds, IEEE *Trans. Automat. Contr.,* Sept. 1997, 42, 1282 – 1286.

*****